DE LA RECHERCHE À L'INDUSTRIE

Infrasound Propagation in Multiple-Scale Random Media Using Generalized Polynomial Chaos

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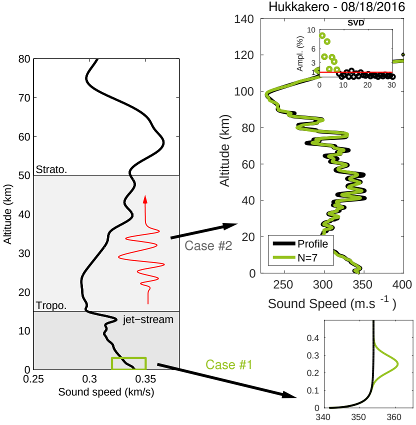
The Atmospheric uncertainties

- Atmospheric Specification (AS):
  - Background state provided by numerical weather forecasts / atmospheric climate reanalysis.
  - Fluctuations of those profiles (like Gavity Waves) are not resolved in ECMWF, G2S, ...
- Uncertainties associated with AS:
  - Can be reduced using SVD decomposition
  - Described by a vector $\xi \in \mathbb{R}^d$
- Assessing the impact of ξ on infrasound/acoustic propagation:
  - Deviation from mean state is large but $d < 10$.
  - For computational model parameters θ, $Y = F(X(\xi); \theta)$ can be represented using a metamodel.



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Outline of the Presentation

1 Polynomial Chaos based Metamodel

– What is a Metamodel ?
– The Polynomial Chaos (gPC) Framework

2 Planetary Boundary Layer (PBL) as a Validation Case

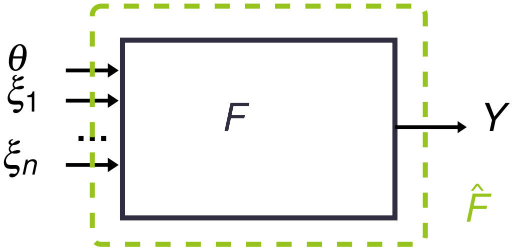
– PBL with a Nocturnal Jet
– Normal Modes Decomposition
– Random Modes and Random Wavepackets
– Performance of the Metamodel

3 Towards a more Realistic Atmosphere

– Incorporating Small-Scale Fluctuations
– Possible Use of the Metamodel

On the use of a Metamodel

Build a metamodel $\hat{F}$ of $Y = F(X(\xi);\theta)$ where $\xi \sim \mathcal{N}(0, \mathbb{I}_n)$:



- Characteristics of a metamodel:
  - Calibrated using a small number of runs of the expensive code.
  - Easy to assess numerically.
  - Reproduce the statistical properties of the output.
- Application to infrasound propagation:
  - Forward uncertainty propagation: Y is a signal at a given distance R
  - Association or localization: Y is a set of by-products (duration, amplitude, ...)

Polynomial Chaos decomposition

A non-intrusive metamodel of $Y = F(X(\xi);\theta)$ where $\xi \sim \mathcal{N}(0, I_n)$

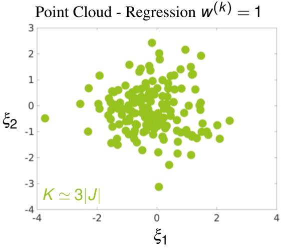
- Polynomial chaos decomposition:

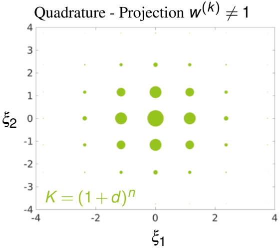
  $$Y = \sum_{j \in J} a_j H_j(\xi) \text{ and } a_j = \langle Y, H_j \rangle,$$

  - $(H_j)_{j \in J}$ set of polynomials (up to degree d)
  - $(H_j)_{j \in J}$ orthonormals for inner product $\langle f, g \rangle = \mathbb{E}[fg]$.

- Cross-validation for order ($|J|$) selection: Leave-One-Out procedure.

- Computation of coefficients $(a_j)_{j \in J}$





$$(a_j)_{j \in J} = \underset{a \in \mathbb{R}^{|J|}}{\arg\min} \| Y(\xi) - \sum_{j \in J} a_j H_j(\xi) \|_2$$

$$\forall j \in J, a_j = \sum_{k=1}^{K} Y^{(k)} H_j(\xi^{(k)}) w^{(k)}$$

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Planetary Boundary Layer as a Validation Case

- Nocturnal jet seen as a perturbation:
  - PBL model from Waxler 2008* + random nocturnal jet u_J:

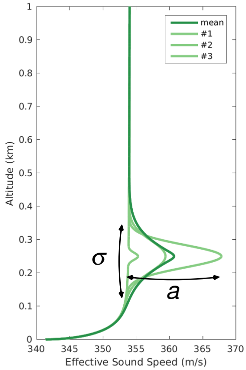
$$u_J(z,\xi) = a \times e^{\frac{z-z_J}{\sigma^2}}$$

$\rightarrow$ Effective celerity: $c(z,\xi) = c_0(z) + u_J(z,\xi)$

  - Uncertainties on the jet properties:

$$\left.\begin{array}{l} a \sim N(m_a, s_a) \\ \sigma \sim N(m_\sigma, s_\sigma) \end{array}\right\} \Rightarrow \xi = (a,\sigma) \in \mathbb{R}^2$$

- Numerical setup:
  - Wave propagation with normal modes (FLOWS).
  - Perfectly Matched Layer used at $z \rightarrow \infty$
  - Neumann homogeneous condition at the ground.
  - Std of the parameters $\rightarrow\sim$ 7% of fluctuation on the profil.



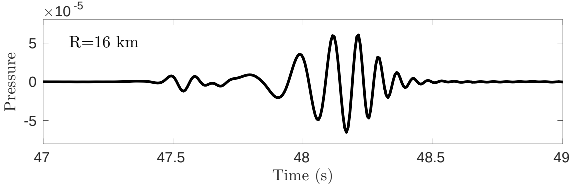
* Waxler *et al.*, JASA, 2008; Chunchuzov *et al.*, JASA, 1990, 2005, Wilson *et al.*, JASA, 2015.

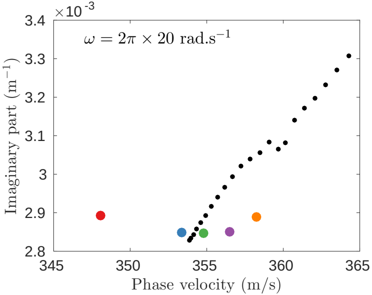
- Wave equation: $H\Psi = k^2\Psi$: $\sigma(H) = \sigma_{\text{disc}}(H) \oplus \sigma_{\text{cont}}(H)$
- Green function at distance R.

$$G(\omega, R) \sim \underbrace{\sum_{l=1}^{N} \frac{\Psi_l(\omega)^2 e^{ik_l(\omega)R}}{\sqrt{k_l(\omega)R}}}_{G_l(R,\omega)}$$

N depends on ω

- Signals and wavepackets.





$$\mathscr{F}^{-1}\left[\sum_{l=1}^{N_0} G_l s_0\right] = \sum_{l=1}^{N_0} \mathscr{F}^{-1}[G_l s_0]$$

$$N_0 = \max N(\omega)$$

- Random medium: $c(z)$ gives $(k_l, \Psi_l(z))$

The acoustic modes

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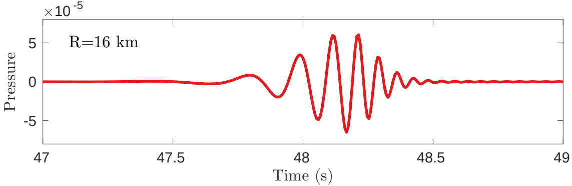

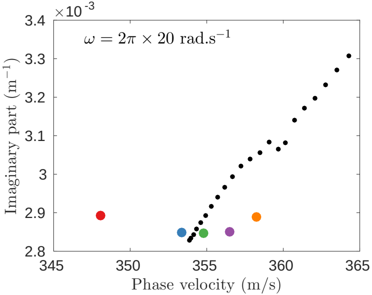

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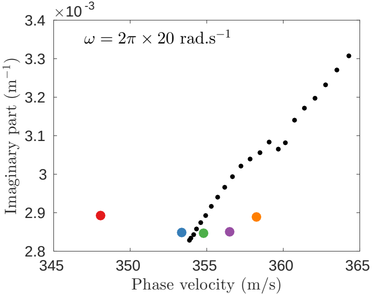

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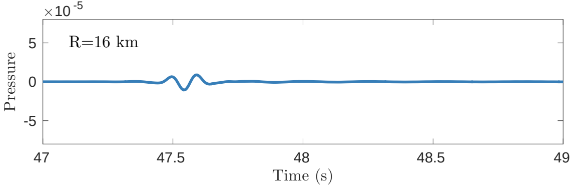

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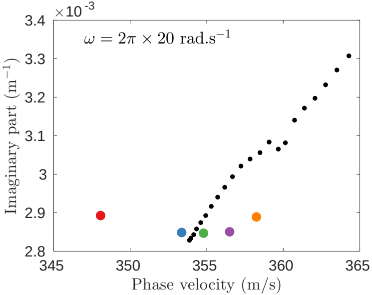

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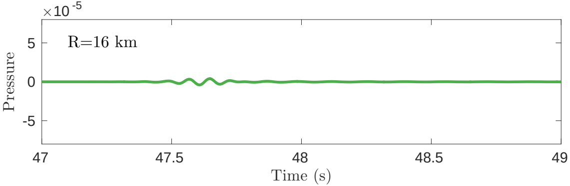

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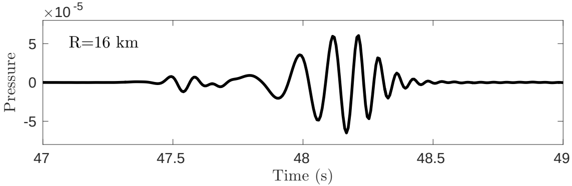
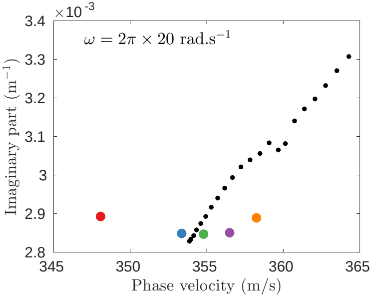
N depends on ω and ξ

- Signals and wavepackets.





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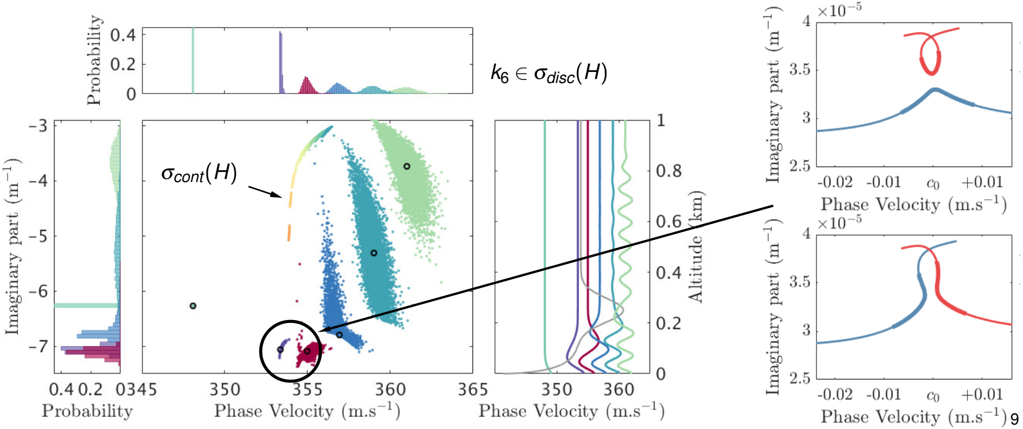
$$N_0 = \max N(\omega, \xi)$$

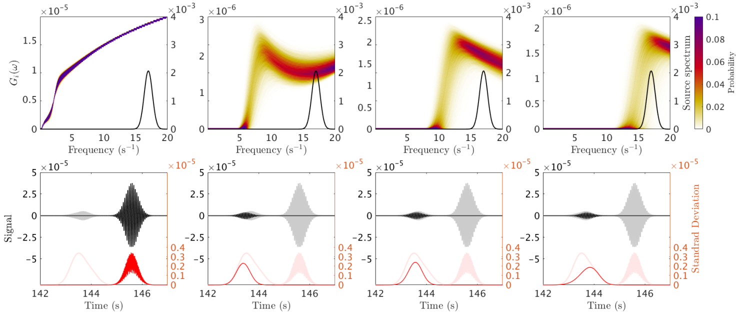
- Random medium: $c(z,\xi)$ gives $(k_l(\xi), \Psi_l(z,\xi))$

- The spectrum of $H(\xi)$ consists in random variables $k_l(\xi)$ that may be difficult to follow as ξ varies:



Random Wavepackets

- PDF of $G_i(\omega, \xi)$ as a function of ξ depends on the mode number i.
- Source is fixed and deterministic, we only look at variability generated by the random medium.

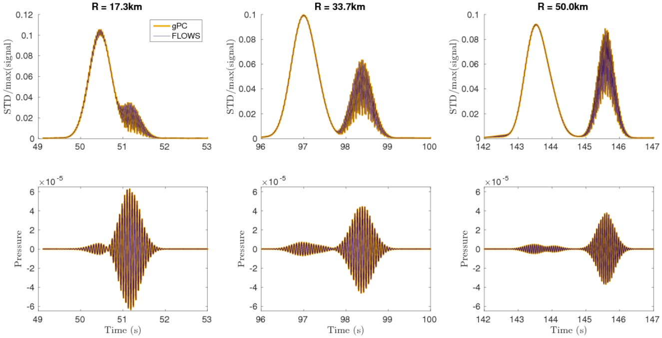


- gPC expansion for $Y = (k_l(\xi), \Psi_l(z, \xi))_{l=1,\dots,N}$:
  - $\tilde{k}_l(\xi) = \sum_{j \in J} a_j^{k_l} H_j(\xi)$
  - $\tilde{\Psi}_l(z, \xi) = \sum_{j \in J} a_j^{\Psi_l}(z) H_j(\xi)$

$\Rightarrow$ Metamodel for $s(t)$.

Convergence properties

- About convergence of signals $\tilde{s}(t,\xi)$ produced by the gPC-metamodel.
  - (1) gPC provides convergence in L^2-norm and (2) G depends continuously on k_l and Ψ_l. Hence $||\tilde{G} - G||_2 \xrightarrow{P\to\infty} 0$.
  - $\mathscr{F}$ is an L^2-isometry and thus, $||\tilde{s}(t,\xi) - s(t,\xi)||_2 \xrightarrow{P\to\infty} 0$.
- Comparison of gPC and Monte-Carlo simulations:



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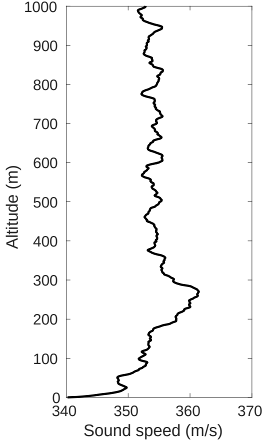
Impact of small-scale fluctuations

- Atmospheric perturbations include large deviation and turbulent noise: $c(z,\xi) = c_0(z,\xi) + \varepsilon c_1(z)$:
- The impact of the turbulent noise can be modelled using the coupling matrix $(C_{nm})_{n,m}$

$$p \sim \sum_{n=1}^{N} \frac{\phi_{n0}^2(\xi) e^{ik_{n0}(\xi)r}}{\sqrt{Rk_{n0}(\xi)}} \left(1 + i\sqrt{\varepsilon} R \frac{\omega^2 C_{nn}(\xi)}{2k_{n0}(\xi)}\right) + \sum_{n=1}^{N} \sum_{m=1}^{N} F((\xi))$$

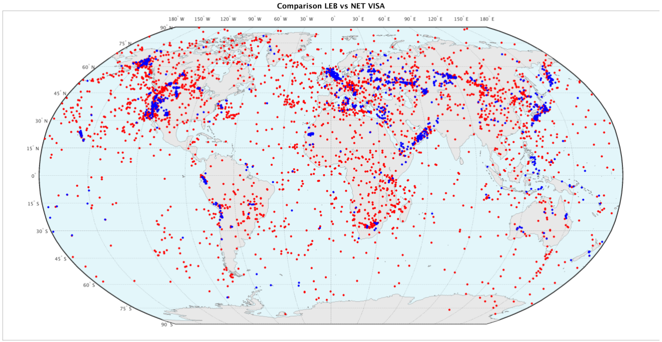
- The coupling matrix depends on the large scale perturbations, its gPC expansion can be derived from $\phi_{n0} = \sum_{k=0}^{+\infty} \alpha_k^{(n)}(z) H_k(\xi)$:

$$\boxed{C_{nl} = \sum_{k=0}^{+\infty} \gamma_k^{(nl)} H_k(\xi)} \text{ where } \gamma_p^{(nl)} = \sum_{j,k} \langle \frac{\mu(z)\alpha_j^{(n)}(z)}{c_0^2(z)}, \alpha_k^{(l)}(z) \rangle \mathbb{E}\left[H_j H_k H_p\right]$$



Take-Home Messages

- gPC framework is an efficient way to obtain a metamodel for complex signals unless $d \gg 10$.
- One metamodel per IMS station provides a way for localization and association procedures, provided a first guess is available.
- Since the metamodel does not depend on source signal it can be used together with a random incoming signal (e.g. Microbarom) with no supplementary cost.



Comparison map of association by NET-VISA (in blue) and by the *Late Event Bulletin* (in red)