DE LA RECHERCHE À L'INDUSTRIE

Métamodèle multi-échelle pour la propagation acoustique en milieu aléatoire

Journée des doctorants de la DIF, 19 juin 2019

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Atmospheric Specification (AS):

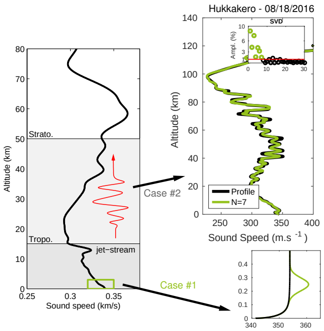
- Background state provided by numerical weather forecasts / atmospheric climate reanalysis.
- Fluctuations of those profiles (like Gavity Waves) not resolved in ECMWF, G2S, ...

Uncertainties associated with AS:

- Can be reduced using SVD decomposition.
- Described by a vector $\xi \in \mathbb{R}^d$.

Assessing the impact of ξ on infrasound:

- Deviation from mean state (due to randomness) is large but $d < 10$.
- For computational model parameters θ, $Y = F(X(\xi); \theta)$ can be represented using a metamodel.



Outline of the Presentation

1 Polynomial Chaos based Metamodel

– What is a Metamodel ?
– The Polynomial Chaos (gPC) Framework

2 The PBL as a Validation Case

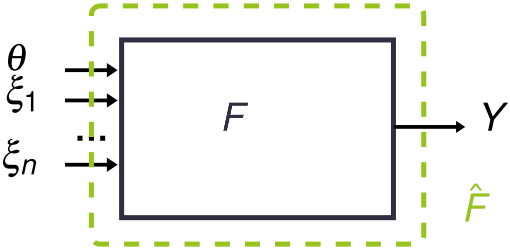
– PBL with a Nocturnal Jet
– Normal Modes Decomposition
– Random Modes and Random Wavepackets
– Signals from the Metamodel

3 Towards a more Realistic Atmosphere

– Incorporating Small-Scale Fluctuations
– Possible Use of the Metamodel

On the use of a Metamodel

Build a metamodel $\hat{F}$ of $Y = F(X(\xi);\theta)$ where $\xi \sim \mathcal{N}(0,\mathbb{I}_n)$:



- Characteristics of a metamodel:
  - Calibrated using a small number of runs of the expensive code.
  - Easy to assess numerically.
  - Reproduce the statistical properties of the output.
- Application to infrasound propagation:
  - Forward uncertainty propagation: Y is a signal at a given distance R.
  - Association or localization: Y is a set of QoIs* (duration, etc.).

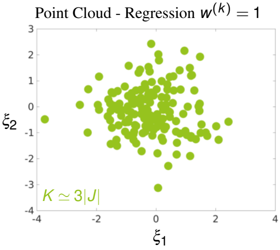
* Quantity of Interest.

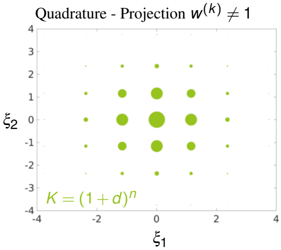
A non-intrusive metamodel of $Y = F(X(\xi);\theta)$ where $\xi \sim \mathcal{N}(0, I_n)$

- Polynomial chaos decomposition:

$$Y = \sum_{j\in J} a_j H_j(\xi) \text{ and } a_j = \langle Y, H_j\rangle,$$

  where $(H_j)_{j\in J}$ is a set of polynomials (up to degree d) and $(H_j)_{j\in J}$ are orthonormals for inner product $\langle f, g\rangle = \mathbb{E}[fg]$.
- Cross-validation for order selection ($|J|$): Leave-One-Out procedure.
- Computation of coefficients $(a_j)_{j\in J}$





$$(a_j)_{j\in J} = \arg\min_{a\in\mathbb{R}^{|J|}} \| Y(\xi) - \sum_{j\in J} a_j H_j(\xi)\|_2 \qquad \forall j \in J, a_j = \sum_{k=1}^{K} Y^{(k)} H_j(\xi^{(k)}) w^{(k)}$$

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Nocturnal jet seen as a perturbation:

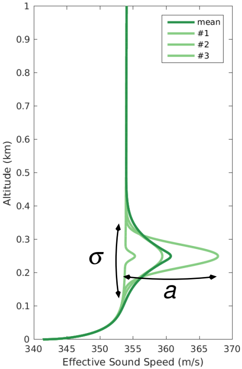
- PBL model from Waxler 2008* + random nocturnal jet u_J at fixed altitude:

$$u_J(z,\xi) = a e^{(z-z_J)/\sigma^2}$$

$\longrightarrow$ Effective celerity: $c(z,\xi) = c_0(z) + u_J(z,\xi)$.

- Uncertainties on the jet properties:

$$\left.\begin{array}{l} a \sim \mathcal{N}(m_a, s_a) \\ \sigma \sim \mathcal{N}(m_\sigma, s_\sigma) \end{array}\right\} \Rightarrow \xi = (a, \sigma) \in \mathbb{R}^2.$$



Numerical setup:

- Wave propagation with normal modes (FLOWS).
- Perfectly Matched Layer used at $z \to \infty$.
- Neumann homogeneous condition at the ground.
- Std of the parameters $\simeq 7\%$ of fluctuation on the profil.

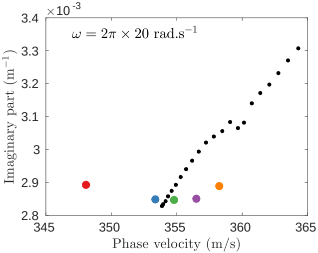
* Waxler *et al.*, JASA, 2008; Chunchuzov *et al.*, JASA, 1990, 2005, Wilson *et al.*, JASA, 2015.

The acoustic modes

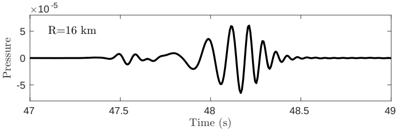
- Wave equation: $H\Psi = k^2\Psi$:
  $\sigma(H) = \sigma_{\text{disc}}(H) \oplus \sigma_{\text{cont}}(H)$
- Green function at distance R.

$$G(\omega, R) \sim \sum_{l=1}^{N} \underbrace{\frac{\Psi_l(\omega)^2 e^{ik_l(\omega)R}}{\sqrt{k_l(\omega)R}}}_{G_l(R,\omega)}$$

  N depends on ω



- Signals and wavepackets.



$$\mathscr{F}^{-1} \sum_{l=1}^{N_0} G_l s_0 = \sum_{l=1}^{N_0} \mathscr{F}^{-1}[G_l s_0]$$
$$N_0 = \max N(\omega)$$

- Random medium: $c(z)$ gives $(k_l, \Psi_l(z))$

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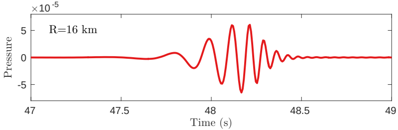

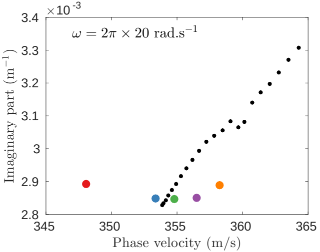

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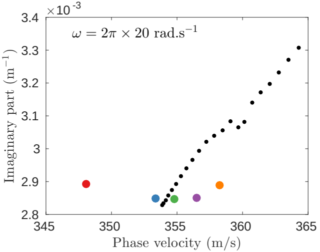

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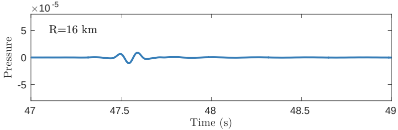

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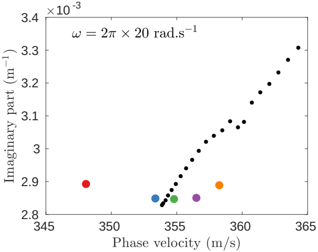

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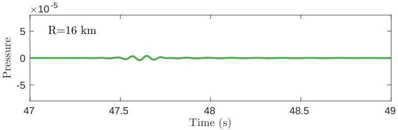

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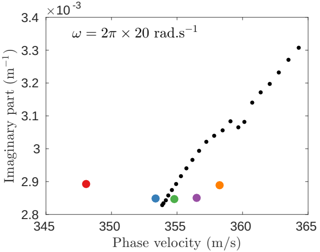
N depends on ω and ξ



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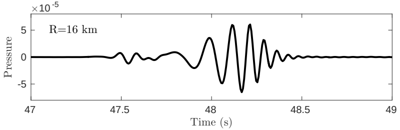

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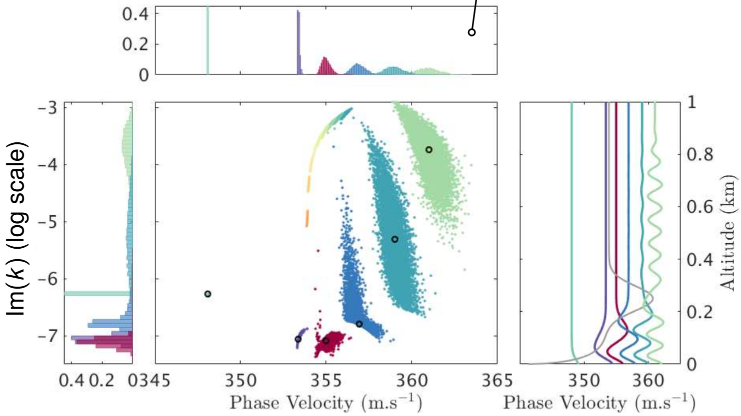
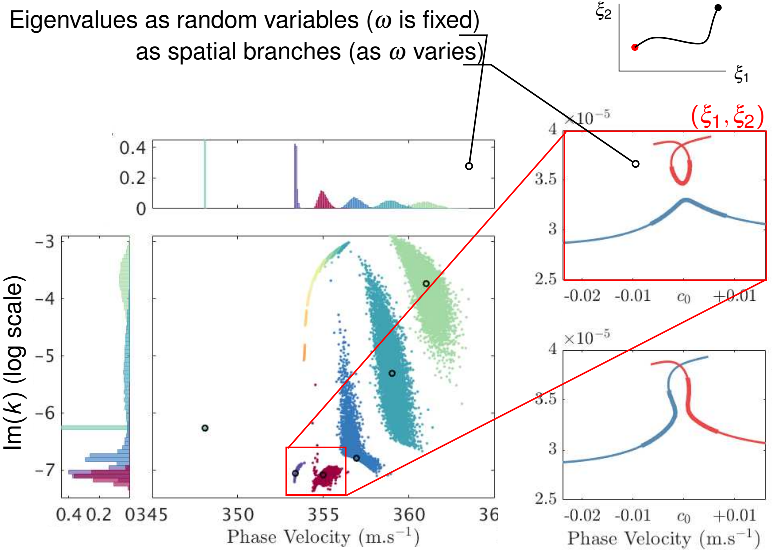
$$N_0 = \max N(\omega, \xi)$$

- Random medium: $c(z,\xi)$ gives $(k_l(\xi), \Psi_l(z,\xi))$

Eigenvalues as random variables (ω is fixed)



Eigenvalues as random variables (ω is fixed)
as spatial branches (as ω varies)



Random Modes

Eigenvalues as random variables (ω is fixed)
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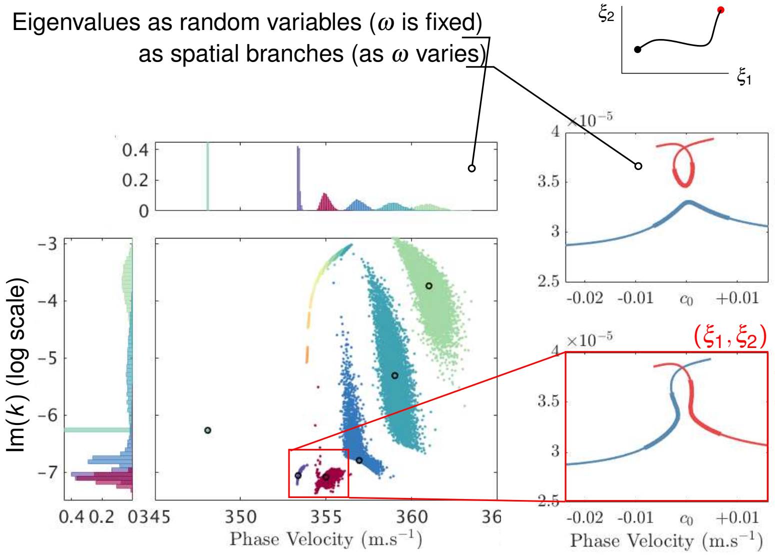


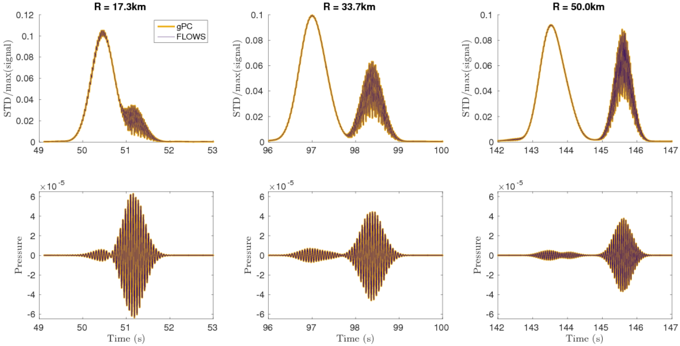
- gPC expansion (= metamodel) for $Y = (k_l(\xi), \Psi_l(z,\xi))_{l=1,\dots,N}$:

$$\tilde{k}_l(\xi) = \sum_{j \in J} a_j^{k_l} H_j(\xi) \text{ and } \tilde{\Psi}_l(z,\xi) = \sum_{j \in J} a_j^{\Psi_l}(z) H_j(\xi)$$

- We deduce the PDF of $G_l(\omega,\xi)$ and thus, a metamodel for the signal:

$$\tilde{s} = \mathscr{F}^{-1} \sum_l \tilde{G}_l s_0 \Rightarrow \mathbb{E}(\tilde{s}), \dots$$

- Comparison with direct Monte-Carlo simulations:



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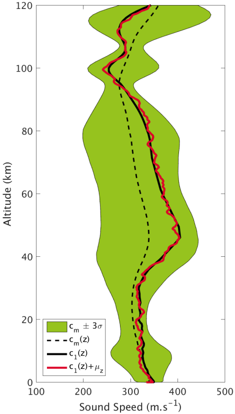
- Atmospheric perturbations include large deviation and turbulent noise:

$$c(z,\xi) = \underbrace{c_0(z,\xi)}_{\text{large}} + \underbrace{c_1(z)}_{\text{small}} = c_0(1+\varepsilon\mu).$$



- The impact of small-variance turbulence is modelled using the coupling matrix $(C_{nm})_{n,m}$

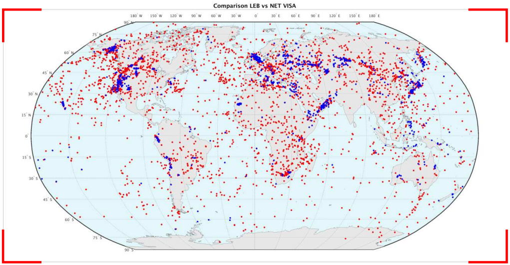
$$p \sim \sum_n G_n(\xi)\left[1 + i\sqrt{\varepsilon}R\frac{\omega^2 C_{nn}(\xi)}{2k_{n0}(\xi)}\right] + \sum_{n,m} D_{nm}(\xi)$$

where D_{nm} is obtained from C_{nm}.

- gPC expansion of C_{nm} (which depends on c_0) can be derived from $\phi_{n0} = \sum_{k=0}^{+\infty} \alpha_k^{(n)}(z)H_k(\xi)$:

$$\boxed{C_{nl} = \sum_{k=0}^{+\infty} \gamma_k^{(nl)} H_k(\xi)} \text{ where } \gamma_p^{(nl)} = \sum_{j,k}\left\langle \frac{\mu(z)\alpha_j^{(n)}(z)}{c_0^2(z)}, \alpha_k^{(l)}(z)\right\rangle \mathbb{E}\left[H_j H_k H_p\right]$$

Perspectives



Bayesian association (NET-VISA) without propagation model
Global Associator (GA) used at IDC

Perspectives at NDC/IDC.

- One metamodel per IMS station to account for propagation effects in operational products (localization, association, ...).
- Goal: advanced (real time) statistical analysis of worldwide detections using (meta)models as constraints (actually 94% of detections are false alarms!).

How can we obtain plausible atmos. specif.?

- Naive approach: use a stochastic process. For small perturbations (less than 0.5%) a perturbative approach is sufficient[1]. But realistic μ is typically $\approx 10\%$.
- Since μ is not stationary (uncertainties propagate!), need to generate stochastic GW[2] fields 'compatible' with climate.

Sanity check: comparison of environmental uncertainty and numerical precision[3].

[1] M. Bertin, Post-doc, DASE, 2016-2018.
[2] B. Ribstein, Post-doc, DASE, 2016-2018.
[3] N. Demeure, Thèse en cours, DSSI/ENS Cachan.