



Did You Know ?

- Atmospheric Specifications (AS) are used to simulate propagation but do not take into account all the variability, for instance Gravity Waves (GW) breaking.
- Gravity waves produce random perturbations of great amplitude (up to 30m/s).

1. Characteristics of Atmospheric Uncertainties

The numerical weather forecasts produce a background state which can be used for infrasound propagation .

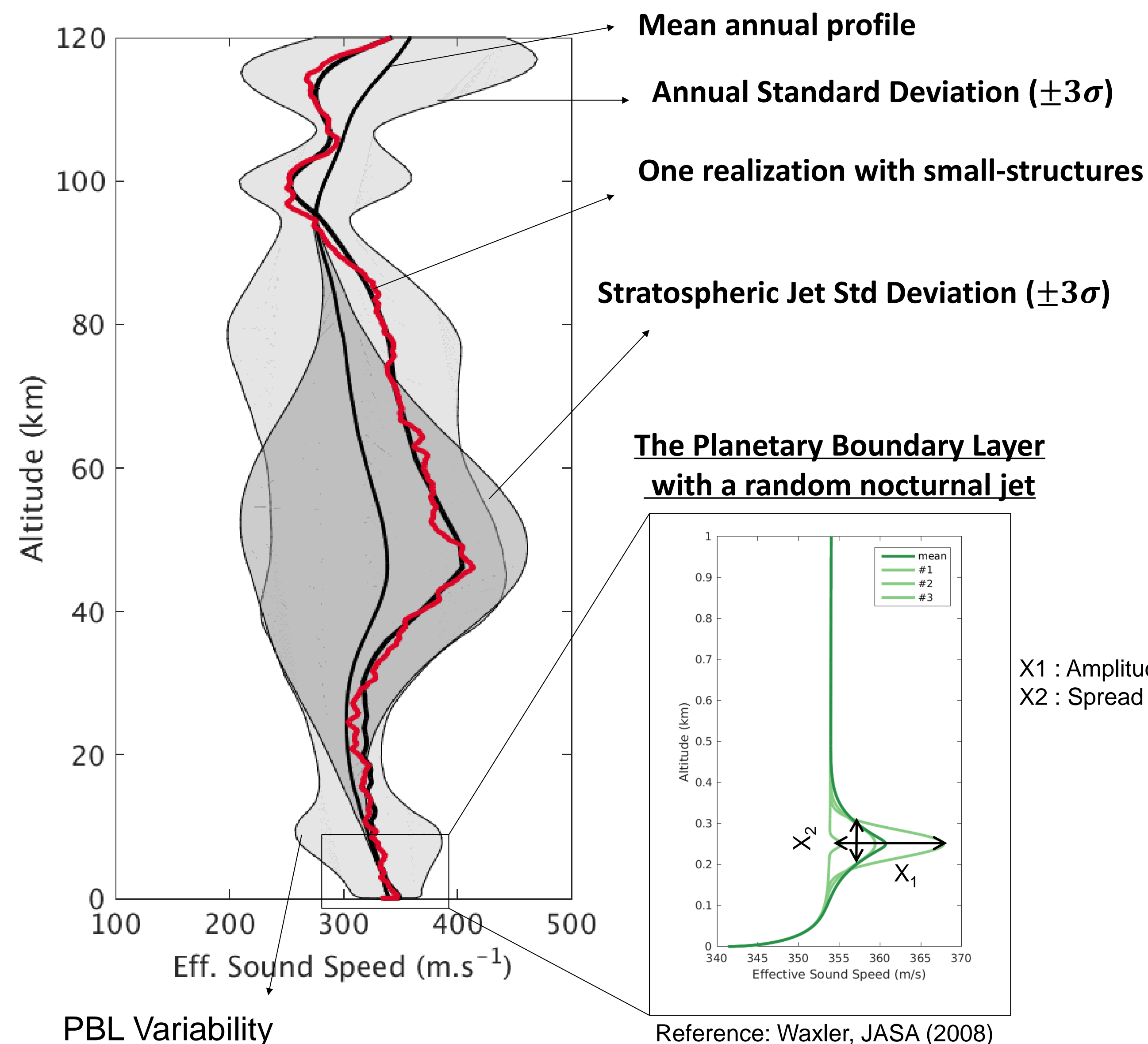
However, the provided temperature and wind profiles do not include small scale fluctuations (e.g. atmospheric turbulence) nor the Gravity Waves propagation.

To assess the impact of those perturbations on acoustic propagation we propose to study two situations :

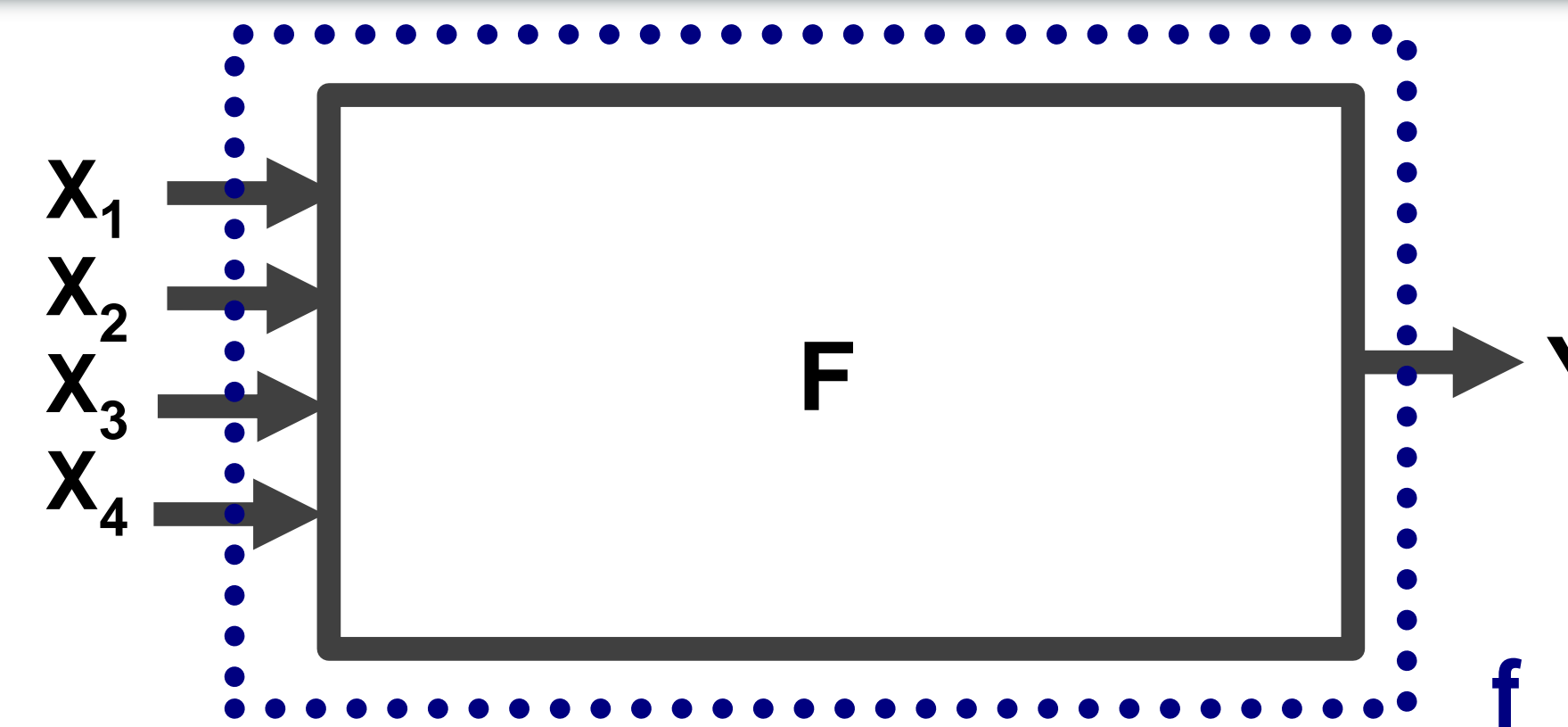
- A random nocturnal jet in a planetary boundary layer : toy model which allows Monte-Carlo comparisons thanks to the size of its domain.
- The impact of the annual variability of the stratospheric jet on infrasonic propagation. (Microbarometric application)

In both cases the amplitude of the perturbation is too large for a perturbative theory to be applied. But because those perturbations can be parameterized with a small number of random parameters the generalized polynomial chaos (gPC) framework is particularly well suited.

Annual variability of a effective sound speed profile



2. Numerical design for a Metamodel with Polynomial Chaos:



Propagation model (F) can be expensive to assess numerically. In order to conduct statistical studies we propose to build a metamodel (f) able to reproduce the statistics of the expensive model with only a small number of run.

To this purpose we use Polynomial Chaos expansion: it consists in developing on a random polynomial basis the output Y as a random function of the input $X=(X_1, X_2, \dots)$:

$$Y = \sum_{j \in J} a_j H_j(X)$$

The coefficients of this expansion can be computed with mainly two methods:

- Regression:

$$(a_j)_{j \in J} = \operatorname{argmin}_{a \in \mathbb{R}^{|J|}} \|Y(X) - \sum_{j \in J} a_j H_j(X)\|_2$$

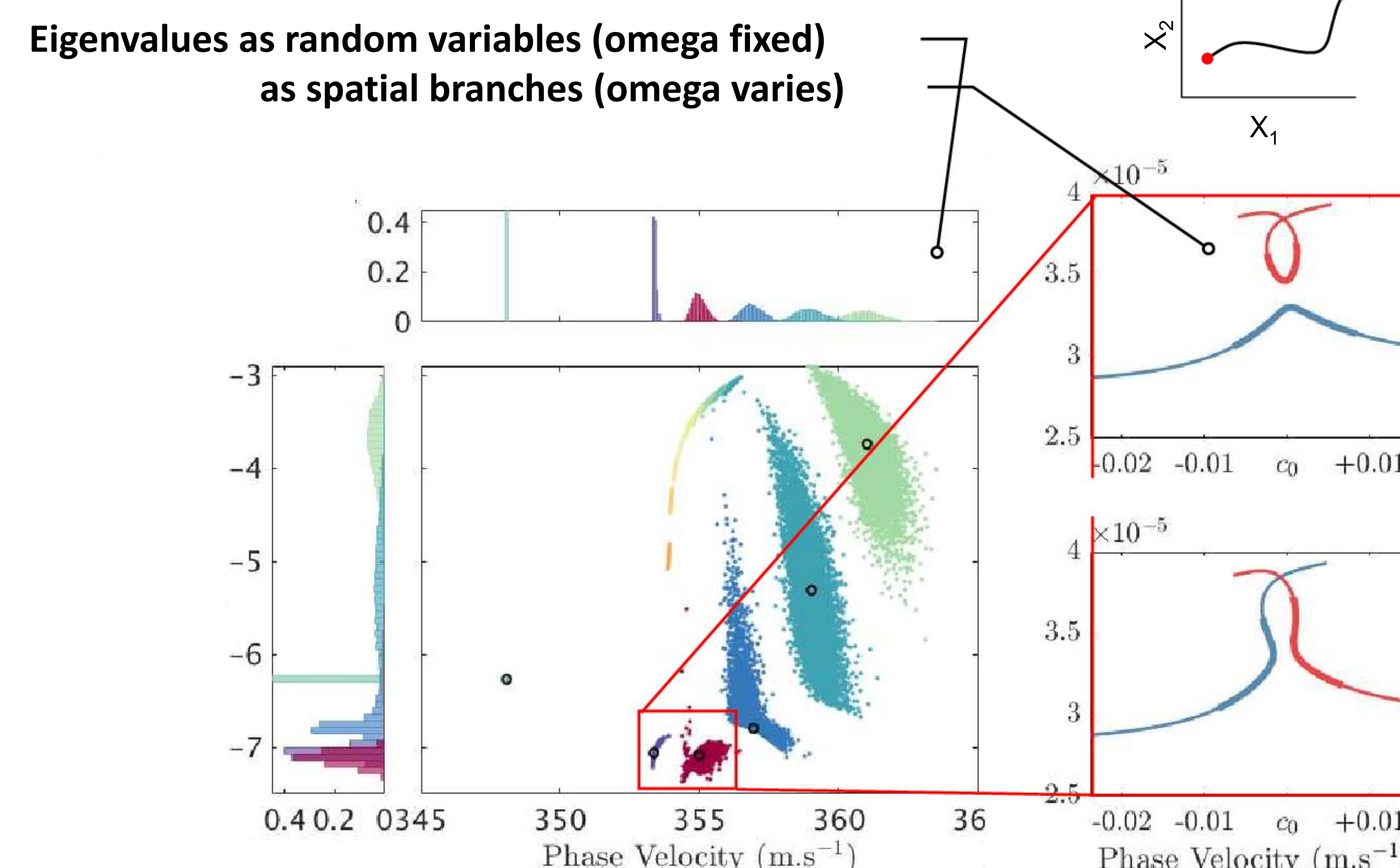
- Projection:

$$\forall j \in J, a_j = \sum_{k=1}^K Y^{(k)} H_j(X^{(k)}) w^{(k)}$$

3. Polynomial Chaos Expansion of the acoustic Modes :

For a random effective sound speed profile, normal modes (i.e. eigenpairs) of the wave equation can be expanded using gPCs.

Random eigenvalues of a toy model for Planetary Boundary Layer :



4. A Metamodel Able to Generate Signals:

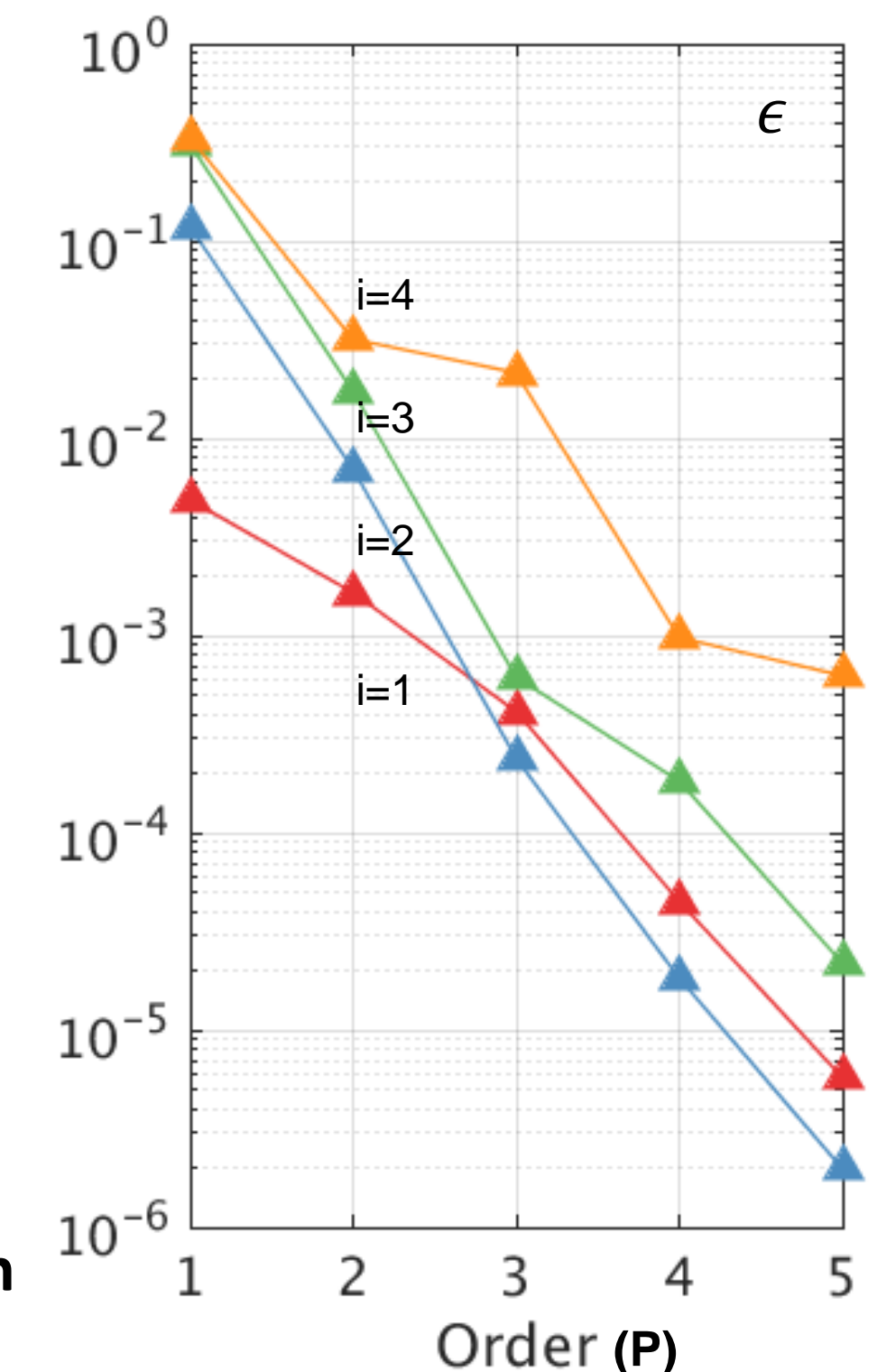
Once the gPC expansions of eigenfunctions and eigenvalues have been computed they can be used to generate wavepackets $s_i(t)$ and deduce signals:

$$signal = \sum_i s_i(t) = \sum_i f \left(\sum_{j=1}^P a_j^i H_j(X) \right)$$

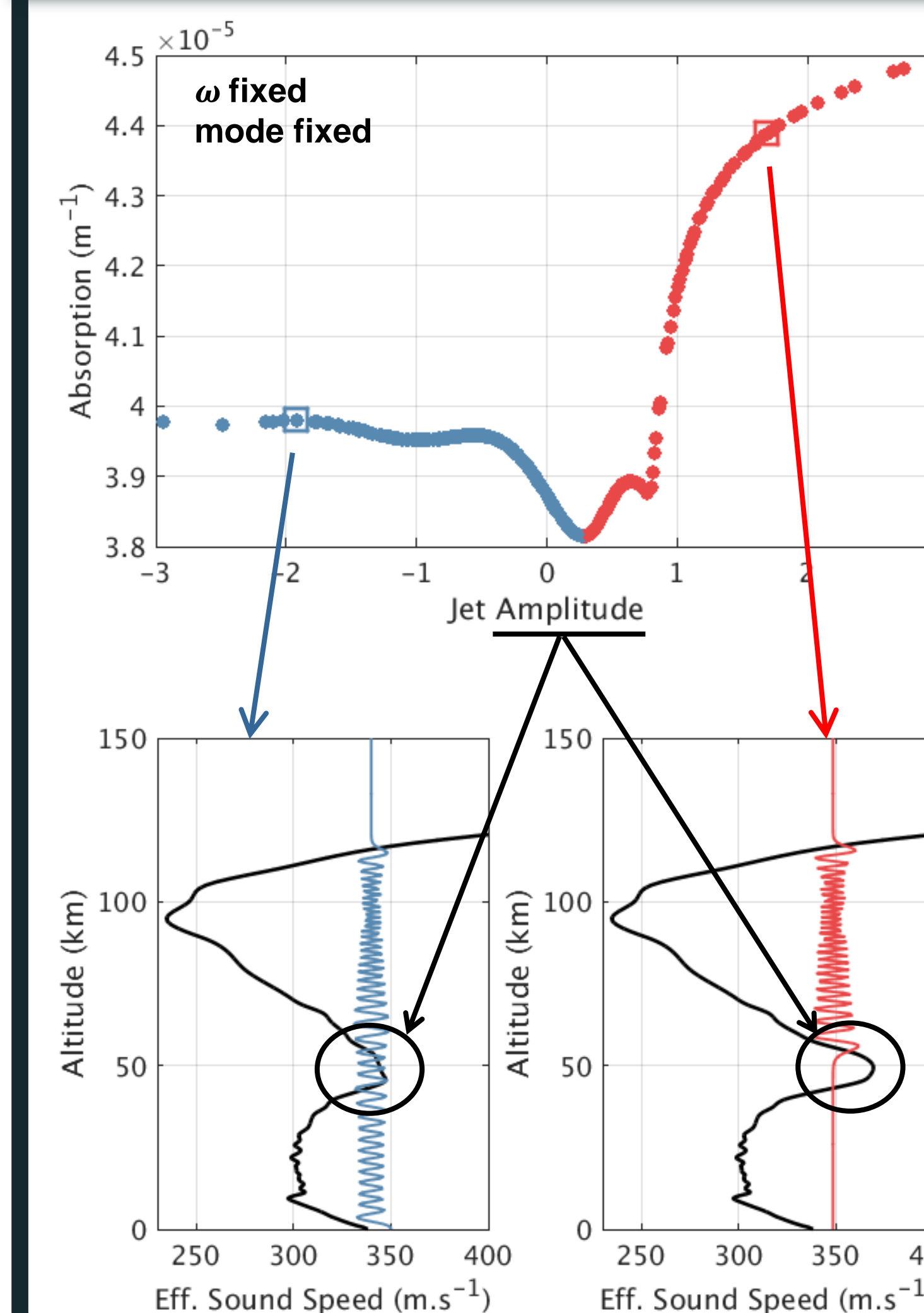
Comparison between gPC (200 real.) and Monte-Carlo (5000 real.) in the case of the PBL. The convergence is evaluated for each wavepacket :

$$\epsilon = \max_t \left(\frac{1}{N} \sum_{i=1}^N |s_i^{gPC}(t) - s_i^{MC}(t)|^2 \right)$$

Naturally, the precision of this expansion increases with the maximal polynomial order (P).



5. Different Scales but the Same Mathematical Problem:



The modes interact with the sound speed profile and the uncertainty can lead to different physical configurations for the same mode.

This dependence requires an ability to follow the eigenvalues depending on the realization and when the frequency evolves.

This different regimes can affect the convergence of the gPC expansion and a particular approach has been developed to tackle this problem.

In fact, the metamodel has to capture those different regimes to ensure a good representation of the variability of the different arrivals in the received signal at the ground.

Conclusion :

- The metamodel is able to reproduce statistics of the signals with a good accuracy and a reduced computational cost.
- The expansion allow to take into account any kind of sources (including random ones).
- With one metamodel per IMS station : association, localization and inversion operation can take propagation effects into account.