Polynomial chaos expansion for wave propagation

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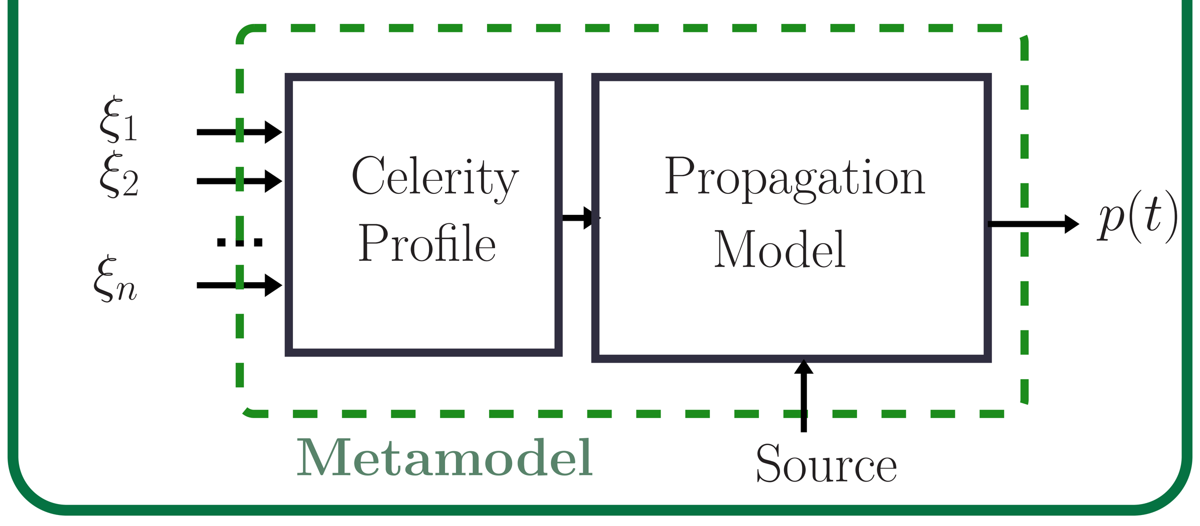
Introduction

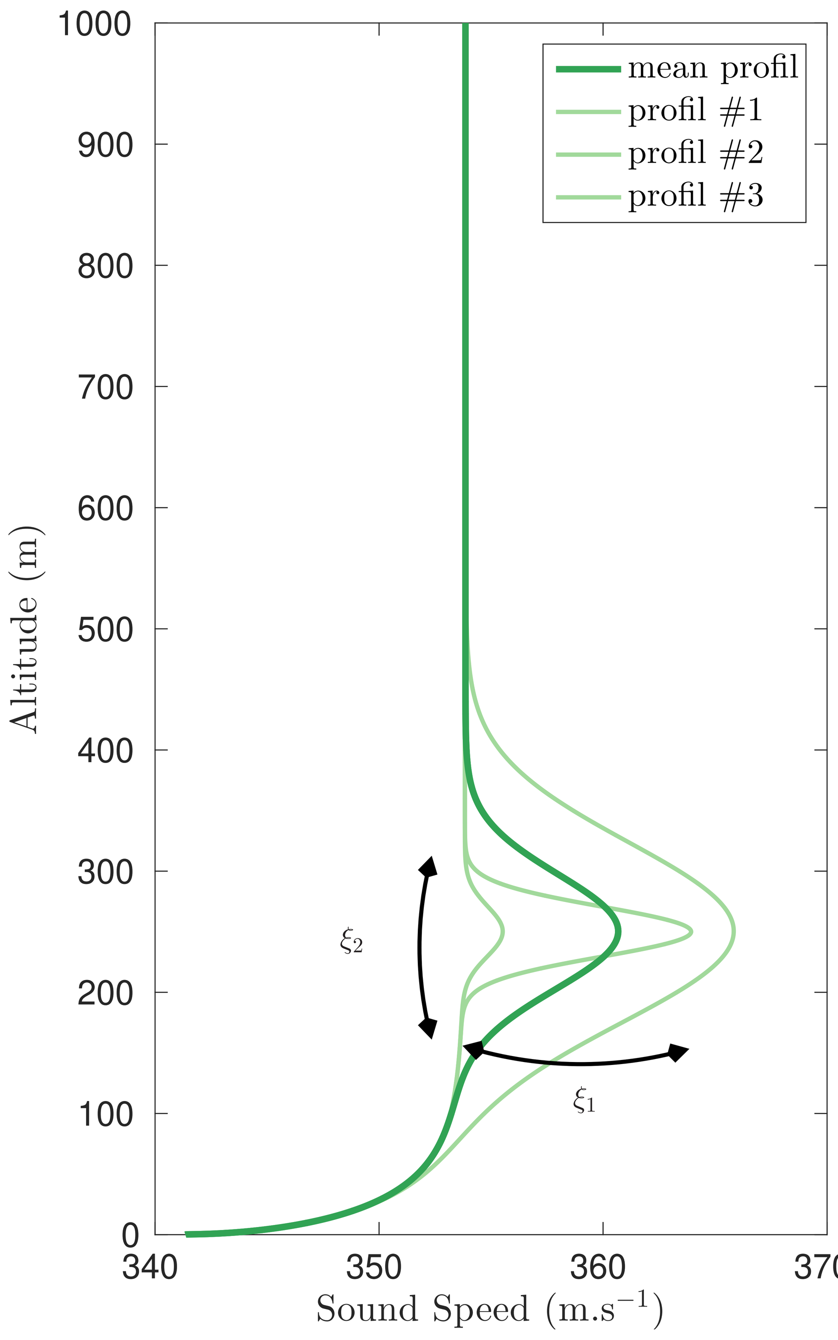
The problem of wave propagation through a random medium arises naturally in many physical applications. The propagation in such a medium results from the superposition of **interfering wave packets**, each one depending on the stochastic characteristics of the medium.
The construction of a metamodel falter over the **long term integration problem** ([1]) which – in this case – appears for long distance propagation. To circumvent this limitation we propose to work in the Fourier domain, and to use a normal mode method to capture the interaction of the propagation with the spatial structures of the medium and of its perturbation.

Framework



Numerical example

Acoustic propagation through a boundary layer profile with nocturnal jet with random parameters.



- ξ_1, ξ_2: **independant** gaussian variables.
- Variance of the parameters: $\sim 7\%$.
- Boundary conditions:
  – Neumann homogeneous at the ground.
  – PML at $z_{max} = 1km$.

References

[1] Xiaoliang Wan and George Karniadakis. Long-term behavior of polynomial chaos in stochastic flow simulations. *Computer Methods in Applied Mechanics and Engineering*, 195:5582–5596, 08 2006.

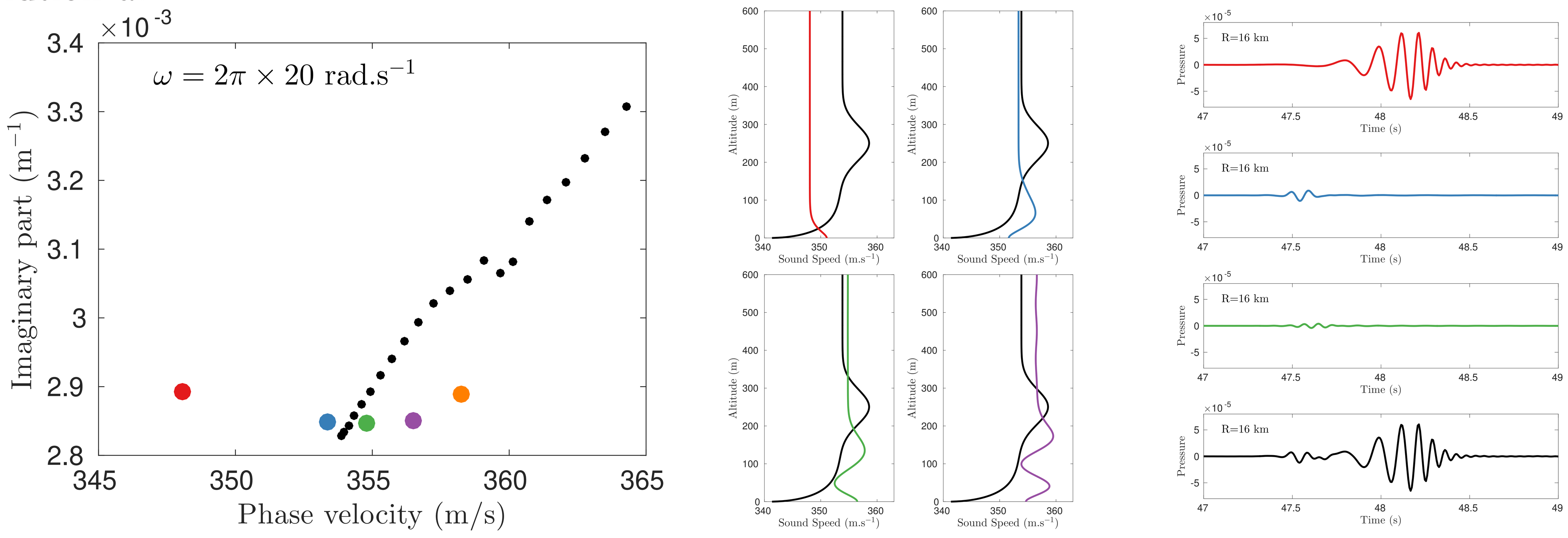
[2] Michael Bertin, Christophe Millet, and Daniel Bouche. A low-order reduced model for the long range propagation of infrasounds in the atmosphere. *The Journal of the Acoustical Society of America*, 136(1):37–52, 2014.

Normal modes decomposition

After a Fourier transform in time, the problem of wave propagation results in solving the Helmoltz equation:

$$\mathcal{H}(x,\xi)u = \Delta u + \frac{\omega^2}{c(x,\xi)^2}u = s(\omega)$$

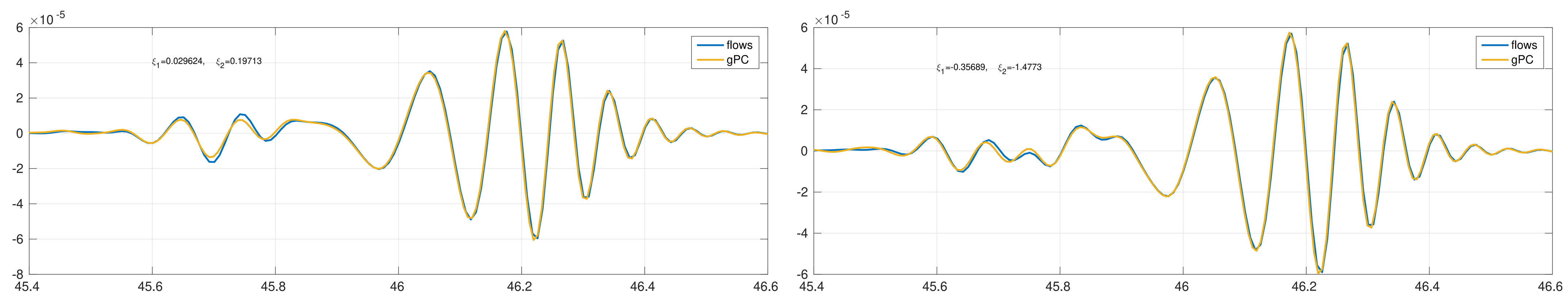
where $s(\omega)$ is the spectrum of the source and $c(x,\xi)$ the wave celerity in the medium. The randomness of the medium gives a wave celerity which depends on random paramaters ξ.
Linear operator theory ensures that **the eigenfunctions** $(\Psi_k)_{k\in K}$ of $\mathcal{H}$ **form a basis** of the space of squarely integrable functions. This basis gives a natural decomposition in wave packets for the solution u.



A modular metamodel

We propose to consider the Polynomial Chaos expansion (gPC) of this basis in order to be able to decompose the solution for every realisation of our medium with a low computational cost. Once the **gPC expansions of the normal modes** $(\widehat{\lambda_k}(\omega,\xi))_{k\in K}$ and $(\widehat{\Psi_k}(x,\omega,\xi))_{k\in K}$ are computed, they can be used to generate signals for a given source at a distance R:

$$\widehat{u}(\omega,R,\xi) = \left[\frac{i}{4}\sum_{k\in K} H_0^{(1)}(\widehat{\lambda_k}(\omega,\xi)R)\widehat{\Psi_k}^2(0,\omega,\xi)\right]s(\omega) \qquad (1)$$



Since the metamodel is built upstream, a stochastic source can be considered without supplementary cost. Sensitivity analysis can also be conducted using those expansions.
Moreover, this appraoch gives a natural framework for **model reduction**: the sum can take into account only the most contributing modes ([2]). For instance, by taking only one mode we have a metamodel for one wave packet which can be usefull when studying a particular arrival in a received signal.

Towards Multi-Level

Atmospheric perturbations include **large deviation** and **turbulent noise**:

$$c(z,\xi) = c_0(z,\xi) + \epsilon c_1(z)$$

Perturbative method allows to take into account the turbulent noise without supplementary cost.

$$u(\omega,R,\xi) = \widehat{u}(\omega,R,\xi) + f(\omega,R,\mathfrak{C}(\xi))$$

where the coupling matrix $\mathfrak{C}(\xi)$ can be developped on the gPC basis.

Conclusion

1. Metamodel able to reproduce signal in a random medium.
2. Metamodel independant from the source, hence the possibility to deal with a stochastic source.
3. Multi-level approach allowing to deal with small structures with reasonable dimension of the input.
4. Modular metamodel which can be used in a context of model reduction.